Holographic Dual of Boundary Conformal Field Theory with Dirichlet Boundary Condition

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based on many works with Chong-Sun

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Outline

1. Background
   - Review of BCFT
   - Boundary Weyl anomaly
   - Holographic BCFT
2. Main Results
   - New Proposal of AdS/BCFT
   - Holographic Stress Tensor
   - Holographic Current
3. Summary and Open questions

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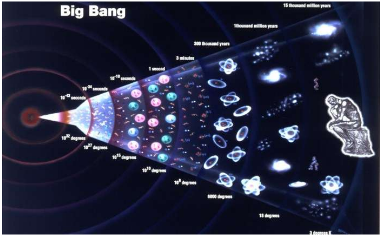
Motivation

Since many physical systems have boundaries, it is interesting to study the boundary effects of quantum systems.

- Casimir effects
- Topological Insulator
- Big Bang of the universe
  It implies that there is a boundary of time.
- Cosmological horizon is also a kind of boundary.



BCFT

- Definitions
  BCFT is a conformal field theory defined on a manifold M with a boundary P, where suitable boundary conditions are imposed.
- Example of free BCFT
  - Conformal free scalar field

$$I = -\frac{1}{2}\int_M d^d x\sqrt{g}[(\partial\phi)^2 + \xi R\phi^2] - \xi\int_P d^{d-1}y\sqrt{\sigma}K\phi^2 \tag{1}$$

  where $\xi = \frac{d-2}{4(d-1)}$, and K is the extrinsic curvature.
  - Conformally invariant boundary conditions

$$\text{Dirichlet BC}: \phi|_P = 0, \tag{2}$$

$$\text{Robin BC}: (\partial_n + 2\xi K)\phi|_P = 0, \tag{3}$$

Outline

Boundary Weyl anomaly

In the presence of boundary, Weyl anomaly of CFT generally pick up a boundary contribution $\langle T^a_a \rangle_P$ in addition to the usual bulk term $\langle T^i_i \rangle_M$, i.e. $\langle T^i_i \rangle = \langle T^i_i \rangle_M + \delta(x_\perp) \langle T^a_a \rangle_P$.

- Bulk contributions to Weyl anomaly

$$\langle T^i_i \rangle_M = \frac{c}{16\pi^2} C^{ijkl} C_{ijkl} - \frac{a}{16\pi^2} E_4 + b\ F_{ij} F^{ij}, \qquad d = 4, \tag{4}$$

- Boundary contributions to Weyl anomaly

$$\langle T^a_a \rangle_P = d_1 \mathrm{Tr} \bar{k}^3 + d_2 C^{ac}{}_{bc} \bar{k}^b{}_a, \quad d = 4, \tag{5}$$

  where $\bar{k}_{ab}$ is the traceless part of extrinsic curvature, C_{ijkl} is the Weyl tensor, F_{ij} is the field strength of gauge field.

- a, b, c are the bulk central charges independent of BC.
- d_i are boundary central charges which depends on BC.

Stress Tensor and Current for BCFT

For BCFT, the stress tensor and current are divergent near the boundary. However, nothing goes wrong since there are boundary contributions, which exactly cancel the bulk 'divergence' and define finite a total energy and charge.

- 'Divergent' stress tensor

$$\langle T_{ij} \rangle = -2\bar{\alpha}_d \frac{\bar{k}_{ij}}{x^{d-1}}, \quad x \sim 0, \tag{6}$$

  where x is the proper distance from the boundary.
- 'Divergent' current

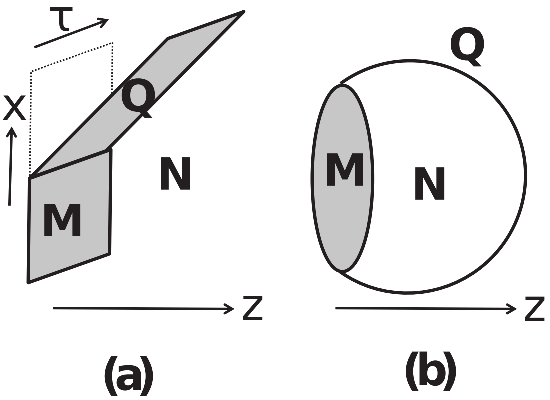
$$< J_i >= \beta_d \frac{F_{in}}{x^{d-3}}, \quad x \sim 0, \tag{7}$$

- It from Weyl anomaly
  Chong-Sun and I find $\bar{\alpha}_d$ and β_d are given by central charges of Weyl anomaly.

Outline

Geometry Setup of AdS/BCFT

The d dimensional manifold M is extended to a $d+1$ dimensional asymptotically AdS space N so that $\partial N = M \cup Q$, where Q is a d dimensional manifold which satisfies $\partial Q = \partial M = P$.



The holographic dual of a half line (a) and a disk (b).

Takayanagi's Proposal of AdS/BCFT

A central issue in the construction of the AdS/BCFT is the determination of the location of Q in the bulk.

- Gravitational action

$$I = \int_N \sqrt{G}(R - 2\Lambda) + 2\int_Q \sqrt{h}(K - T), \tag{8}$$

where T is the holographic dual of BC, since it affects the boundary central charges as the BC does.

- Variation of the action on Q

$$\delta I|_Q \quad = \quad -\int_Q \sqrt{\gamma}\left(K^{\alpha\beta} - (K - T)h^{\alpha\beta}\right)\delta h_{\alpha\beta}. \tag{9}$$

- Takayanagi proposed to impose Neumann BC on Q:

$$K_{\alpha\beta} - (K - T)h_{\alpha\beta}|_Q = 0 \tag{10}$$

to fix the position of Q.

Outline

My Proposal of AdS/BCFT

In general, there are more than one consistent BCs for a theory. I find Dirichlet BC works as well as Neumann BC for holographic BCFT.

- Variation of the action on Q

$$\delta I|_Q = -\int_Q \sqrt{\gamma}\left(K^{\alpha\beta} - (K - T)h^{\alpha\beta}\right)\delta h_{\alpha\beta}. \tag{11}$$

- I propose to impose Dirichlet BC on Q

$$\delta h_{\alpha\beta}|_Q = 0, \tag{12}$$

  to determine the position of Q.
- For simplicity, we fix $h_{\alpha\beta}$ to be AdS metric

$$R^Q_{ijkl} + \text{sech}^2\rho(h_{ik}h_{jl} - h_{il}h_{jk}) = 0, \tag{13}$$

  where $T = (d-2)\tanh\rho$.

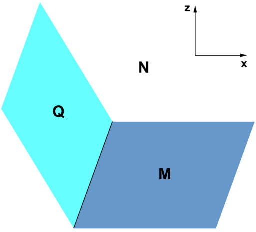
Not all solutions to Einstein equations are allowed in AdS/BCFT. AdS is indeed a solution to holographic BCFT with both NBC and DBC.

Poincare AdS

$$ds^2 = \frac{dz^2 + dx^2 + \delta_{ab} dy^a dy^b}{z^2}, \tag{14}$$

is a solution to both NBC (10) and DBC (12), provided that the embedding function of Q is given by

$$x = -\sinh\rho\, z, \tag{15}$$



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The bulk metric

We aim to find solutions to AdS/BCFT for curved boundaries with extrinsic curvatures.

- Ansatz for the metric

$$ds^2 = \frac{1}{z^2}\Big[dz^2 + dx^2 + \big(\delta_{ab} - 2x\bar{k}_{ab}f(\frac{z}{x}) - 2x\frac{k}{d-1}\delta_{ab}\big)dy^a dy^b + O(k^2$$

- Einstein equations at $O(k)$

$$s\left(s^2+1\right)f''(s) - (d-1)f'(s) = 0, \qquad (17)$$

- Solutions

$$f(s) = 1 + \bar{\alpha}_d \frac{s^d\ {}_2F_1\left(\frac{d-1}{2}, \frac{d}{2}; \frac{d+2}{2}; -s^2\right)}{d}, \qquad (18)$$

where $\bar{\alpha}_d$ are constant to be determined by boundary conditions.

The embeding function of Q

We aim to find solutions to AdS/BCFT for curved boundaries with extrinsic curvatures.

- Ansatz for the embeding function of Q

$$x = -\sinh\rho \; z + \lambda_2 k z^2 + O(k^2) \tag{19}$$

- BC fix the bulk boundary Q
  Imposing either Neumann BC (10) or Dirichlet BC (13), we can fix

$$\lambda_2 = \frac{\cosh^2\rho}{2(d-1)}. \tag{20}$$

- BC fix the bulk metric

$$\bar{\alpha}_{Nd} = \frac{-d\cosh^d\rho}{(-\coth\rho)^d \, {}_2F_1\left(\frac{d-1}{2}, \frac{d}{2}; \frac{d+2}{2}; -\text{csch}^2\rho\right) + d\cosh^2\rho\coth\rho},$$

$$\bar{\alpha}_{Dd} = \frac{-d(-\text{csch}\rho)^{-d}}{{}_2F_1\left(\frac{d-1}{2}, \frac{d}{2}; \frac{d+2}{2}; -\text{csch}^2\rho\right)}. \tag{21}$$

Holographic Stress Tensor

Holographic stress tensor takes the expected form near the boundary.

- Asymptotic AdS in Fefferman-Graham expansion

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}(g^{(0)}_{ij} + z^2 g^{(1)}_{ij} + \cdots + z^d h^{(d)}_{ij} + \cdots)dy^i dy^j. \quad (22)$$

- Holographic stress tensor

$$T_{ij} = d h^{(d)}_{ij} = -2\bar{\alpha}_d \frac{\bar{k}_{ij}}{x^{d-1}} + O(k^2), \quad (23)$$

- Holographic Weyl anomaly
  One can show that $\bar{\alpha}_d$ are indeed the central charges of holographic Weyl anomaly.

Outline

Holographic Current for 4d BCFT

Following a similar approach, we can derive the holographic current.

- BCs for gauge field

$$\begin{aligned} \text{Absolute BC} &: \mathcal{F}_{n\mu}|_Q = 0, \\ \text{Relative BC} &: {}^*\mathcal{F}_{n\mu}|_Q = 0. \end{aligned} \tag{24}$$

- Holographic current

$$< J^i >= \lim_{z\to 0} \frac{1}{\sqrt{g}} \frac{\delta I}{\delta A_i} = -\frac{F^i{}_n}{x} + O(1) \tag{25}$$

- Holographic Weyl anomaly

$$\mathcal{A} = \int_M \sqrt{g}[bF_{ij}F^{ij} + O(R^2)], \quad b = -\frac{1}{4}. \tag{26}$$

- Holographic BCFT obey the universal law for anomaly-induced current

$$< J^i >= 4b\frac{F^i{}_n}{x}, \quad x \sim 0. \tag{27}$$

Exact Result of Holographic Current in 3d

When $T = \rho = 0$, 4D magnetic charged Reissner-Nordström black holes satisfy the Neumann BC for metrics and the relative BC for gauge fields.

- Magnetic charged RN black hole

$$\begin{aligned} \text{metric}: \quad & ds^2 = \frac{dz^2/f(z) - f(z)dt^2 + dx^2 + dy^2}{z^2}, \\ \text{gauge field}: \ & \mathcal{A} = (Bx)dy, \\ \text{Q}: \quad & x = 0, \end{aligned} \tag{28}$$

where $f(z) = 1 - Mz^3 + B^2 z^4/4$.

- Holographic boundary current

$$j_y = Bz_h, \tag{29}$$

where z_h is the location of outer horizon

- The boundary current decreases as the temperature increases

Summary and Outlook

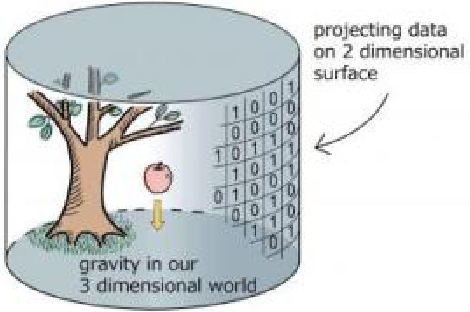
Summary:

- We propose a new model of holographic dual of BCFT based on Dirichlet BC.
- Our new model gives the correct one point function of stress tensor and current.
- Our model predicts that a constant magnetic field in the bulk can induce a constant current on the boundary in three dimensions.

Outlook:

- Holographic BCFT with Conformal BC?
- Boundary Effects of Cosmology and Condensed Matter?

Only one piece of the boundary can reconstract all the apples in the bulk.



Thank you!